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Engineering Notes

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Aeroelastic Model Design Using an Integrated Optimization Approach

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Introduction

TESTING of aeroelastic wind-tunnel models is a common task in aircraft development programs. The objective may be to verify the aeroelastic behavior of a particular aerospace vehicle or to verify aeroelastic analysis, for example, see Baker et al.[1] and Sensburg et al.[2] Often, the aeroelastic behavior of the wind-tunnel model has to represent the behavior of the full-size structure in an aeroelastically scaled sense.[3] Traditionally, the design and manufacturing of aeroelastically scaled wind-tunnel models have been performed by experienced craftsmen and based on a significant amount of empirical knowledge and corrections.

Today, the interest for more straightforward design procedures has increased, and approaches based on numerical optimization have been proposed by, for example, French and Eastep,[4] Amiryants and Ishmuratov,[5] and Eller.[6] The problem of scaled model design has much in common with the task to identify or update an analytical model based on experimental data. Such an update can be treated as a parameter identification problem.[7,8]

This Note is a contribution to the model design aspect of aeroelastic testing. Whereas French and Eastep[4] present a sequential design approach where the static and dynamic properties of the model are handled separately, the present study shows that it may be advantageous to consider both the static and dynamic properties simultaneously in an integrated design procedure.

Design Objective

A half-span aeroelastic wind-tunnel model representing a blended wing body (BWB) aircraft is designed and tested with respect to aeroelastic behavior as earlier reported in Refs. 9 and 10. The model is designed using the segmented approach[11] with an internal carbon fiber/epoxy composite beam as primary structure (Fig. 1). Only the outer part of the model (from $y = 0.45$ m) is elastic, whereas the inner part containing the load balance is rigid.

In this study, a new internal composite wing beam is designed, and additional tuning masses are sized. The design objective is that the model should be representative for the full-size aircraft in terms of both static and dynamic aeroelastic behavior at a certain flight condition. A finite element model of the full size BWB wing structure serves as reference for the model design.[12,13] The model planform is almost exactly scaled compared to the full size reference.[14] A set of reference points on the planform is introduced to provide a set of comparable geometric points on the model and the full size structure, respectively, as shown in Fig. 1.

The model similarity laws and scaling ratios depend on the testing conditions, as well as on the full-size flight condition, as discussed by Bisplinghoff et al.[3] Statically, the stiffness of the model is chosen to show the same relative deformation during testing as the full-size aircraft at the corresponding flight condition. The dynamic similarity laws require that the reduced frequencies and the mode shapes are preserved. Also, the total mass and center of gravity position is to be accurately scaled. The model scaling ratios for the considered model are given in Fig. 1.

A number of reference load cases are applied to the full-size structure, and the deformations are observed in the reference points. In this study, four out-of-plane point forces at different wing locations are used as reference loads. The objective is that the model will experience the same relative displacements when loaded with the same scaled reference loads. The objective for the stiffness of the model can, hence, be expressed in terms of a desired flexibility matrix $\bar{\boldsymbol{\Delta}}$ according to

$$\bar{\boldsymbol{\Delta}} = \begin{bmatrix} | & & | \\ \bar{\delta}_1 & \dots & \bar{\delta}_p \\ | & & | \end{bmatrix} \tag{1}$$

where p is the number of considered reference load cases. Here, $\bar{\delta}_1$ represents the desired out-of-plane displacements of the model evaluated in the reference points when subjected to reference load case 1.

A desired mode shape matrix can be expressed as

$$\bar{\boldsymbol{\Phi}} = \begin{bmatrix} | & & | \\ \bar{\phi}_1 & \dots & \bar{\phi}_q \\ | & & | \end{bmatrix} \tag{2}$$

where $\bar{\phi}_1$ is the first mode shape in terms of displacements at the reference points. Here, q is the number of mode shapes considered

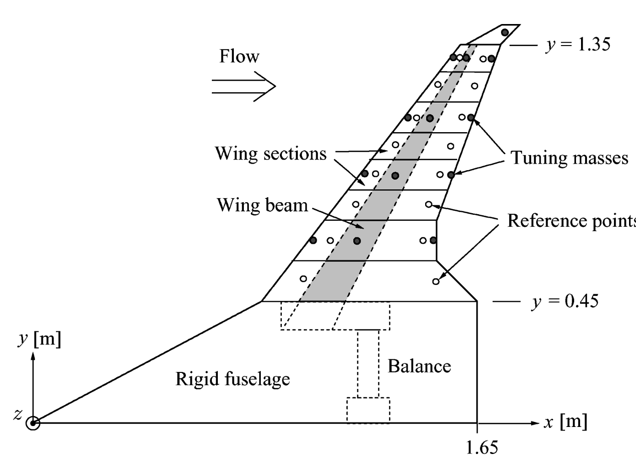

Fig. 1 Wind-tunnel model; model scaling ratios are length 0.0351, air density 2.97, velocity 0.133, frequency 3.80, mass 1.28×10^{-4}, and force 6.49×10^{-5}

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important for the design. Furthermore, a vector of corresponding desired model eigenfrequencies is specified as

$$\bar{\mathbf{\Omega}} = [\bar{\omega}_1 \quad \dots \quad \bar{\omega}_q] \tag{3}$$

In this study, three modes and frequencies are considered. The first two are the first and second bending mode of the wing, and the third is the first torsional mode. However, additional modes and frequencies may be used.

Sequential Design Optimization

When sequential design approach, is used, the design procedure is divided in two steps.[4] In the first step, the stiffness of the model is considered and structural design variables are determined to match the stiffness objective. In the second step, additional tuning masses are sized and distributed inside the model to adjust the natural frequencies and mode shapes.

During the stiffness design phase, an optimization problem based on the desired flexibility matrix $\bar{\mathbf{\Delta}}$ is formulated as

$$\begin{aligned} &\underset{\boldsymbol{x}_s}{\text{minimize}} \quad f_\delta(\boldsymbol{x}_s) \\ &\text{subject to} \quad \underline{\boldsymbol{x}}_s \le \boldsymbol{x}_s \le \bar{\boldsymbol{x}}_s \end{aligned} \tag{4}$$

The objective $f_\delta(\boldsymbol{x}_s)$ is here expressed as

$$f_\delta(\boldsymbol{x}_s) = \sum_{i=1}^{r}\sum_{j=1}^{p}[\delta_{ij}(\boldsymbol{x}_s) - \bar{\delta}_{ij}]^2 \tag{5}$$

where $\boldsymbol{x}_s$ is the vector of structural design variables with lower and upper bounds $\underline{\boldsymbol{x}}_s$ and $\bar{\boldsymbol{x}}_s$, respectively. The objective consists of the least-square sum between all elements of the flexibility matrix for the current design $\delta_{ij}(\boldsymbol{x}_s)$ and the corresponding desired value $\bar{\delta}_{ij}$. In Eq. (5), r is the number of reference displacements for each load case.

When the dynamic properties are concerned, both the frequencies and the mode shapes must be considered during the design. As concluded by French and Eastep,[4] accurate natural frequencies in combination with a good stiffness match do not generally ensure that the mode shapes are correct. Hence, both the frequencies and the mode shapes must be included when formulating the optimization objective. One possibility is to use a weighted objective according to

$$\begin{aligned} &\underset{\boldsymbol{x}_m}{\text{minimize}} \quad k_\omega f_\omega(\boldsymbol{x}_m) + k_\phi f_\phi(\boldsymbol{x}_m) \\ &\text{subject to} \quad \underline{\boldsymbol{x}}_m \le \boldsymbol{x}_m \le \bar{\boldsymbol{x}}_m \end{aligned} \tag{6}$$

Here, k_ω and k_ϕ are weighting factors. The frequency objective $f_\omega(\boldsymbol{x}_m)$ and mode shape objective $f_\phi(\boldsymbol{x}_m)$ are expressed according to

$$f_\omega(\boldsymbol{x}_m) = \sum_{j=1}^{q}\left[\frac{\omega_j(\boldsymbol{x}_m) - \bar{\omega}_j}{\bar{\omega}_j}\right]^2 \tag{7}$$

$$f_\phi(\boldsymbol{x}_m) = \sum_{i=1}^{r}\sum_{j=1}^{q}[\phi_{ij}(\boldsymbol{x}_m) - \bar{\phi}_{ij}]^2 \tag{8}$$

where $\omega_j(\boldsymbol{x}_m)$ and $\bar{\omega}_j$ are the jth frequency of the current design and the desired value, respectively. Here, the difference in frequency is normalized by the desired value to put an equal relative weight to lower and higher frequencies. When the mode shapes are considered, $\phi_{ij}(\boldsymbol{x}_m)$ denotes the ith element of mode number j for the current design. The corresponding desired value is represented by $\bar{\phi}_{ij}$. Note that all modes have to be normalized in a consistent manner to be comparable. During this study, all mode shapes are normalized such that $\|\phi\|_\infty = 1$. During mass design, $\boldsymbol{x}_m$ is the vector of mass design variables. Constraints considering the total mass of the model and center of gravity position are not used for this particular design, but may sometimes be required.

Integrated Design Optimization

An alternative to the sequential design approach is to use an integrated design procedure, where both the structural variables and mass variables are determined simultaneously to balance the static and dynamic objectives. A weighted objective for such an integrated design can be expressed as

$$\begin{aligned} &\underset{\boldsymbol{x}}{\text{minimize}} \quad k_\delta f_\delta(\boldsymbol{x}) + k_\omega f_\omega(\boldsymbol{x}) + k_\phi f_\phi(\boldsymbol{x}) \\ &\text{subject to} \quad \underline{\boldsymbol{x}} \le \boldsymbol{x} \le \bar{\boldsymbol{x}} \end{aligned} \tag{9}$$

where the individual objectives $f_\delta(\boldsymbol{x})$, $f_\omega(\boldsymbol{x})$, and $f_\phi(\boldsymbol{x})$ are given in Eqs. (5), (7), and (8), respectively. Now $\boldsymbol{x} = [\boldsymbol{x}_s, \boldsymbol{x}_m]^T$ is the vector of all design variables including both the structural parameters and the tuning masses. Because this formulation implies a larger design space because all variables can be varied independently, the formulation is also likely to be less convex. Care must also be taken when choosing the values on the weighting factors to get a well-balanced design.

Results

The structural analysis of the wind-tunnel model, in terms of static elastic analysis and modal analysis, is performed using the finite element approach.[15] For numerical optimization, a sequential quadratic programming algorithm is used, where the gradients are obtained using finite differences.[16]

For the present model, eight structural variables representing thickness parameters of the composite beam are used. Two additional structural variables (composite layup angles) are used for controlling the ratio between bending and torsional stiffness, as well as the amount of bending/torsion coupling.[14] For mass balancing, the mass of 13 lead tuning masses (Fig. 1) are used as design variables. In total, this gives 10 structural variables for the static design and 13 mass variables for dynamic tuning, hence, 23 variables in total when using the integrated formulation.

First, the stiffness of the model was considered, and the structural design variables were determined by solving the optimization problems (4) and (5). From an arbitrary initial guess, the design converged rather quickly to a design with negligible deviation from the desired flexibility. In this case, the maximum deviation was about 4% (Table 1). The deviation here refers to a vector of relative deviation $\boldsymbol{d}_j$ associated with each load case (j) defined as

$$\boldsymbol{d}_j = [\boldsymbol{\delta}_j(\boldsymbol{x}) - \bar{\boldsymbol{\delta}}_j]/\|\bar{\boldsymbol{\delta}}_j\|_\infty \tag{10}$$

A similar expression holds for the deviation in mode shape, where the static displacement vector $\boldsymbol{\delta}$ is replaced by the mode shape vector $\boldsymbol{\phi}$.

With the structural design variables fixed, the masses were sized with respect to the dynamic objective according to Eqs. (6–8). The first bending mode of the wing was very well represented by the converged design. Also the torsional mode was very well represented. However, significant difficulties were experienced for the second bending mode of the wing. Various weighting ratios between frequencies and mode shapes, as well as different initial guesses, were tested; nevertheless, the matching was rather poor with a maximum

Table 1 Relative deviations from desired behavior for sequential and integrated design

Design measure	Maximum deviation	
	Sequential, %	Integrated, %
Load case 1	3.1	3.8
Load case 2	3.9	5.6
Load case 3	2.0	2.1
Load case 4	2.4	3.0
Mode 1	0.9	1.3
Mode 2	51.7	5.6
Mode 3	20.3	15.7
Frequencies	1.2	4.1

relative amplitude deviation of about 50% for one of the wing tip reference points (Table 1). The frequencies did not cause any significant trouble, and the agreement overall was very good.

To improve the design, an integrated design optimization was performed according to expression (9). Here, the converged solution from the sequential approach was used as the initial guess. All structural and mass variables were then varied to further improve the model behavior. By careful choice of the three weighting factors, it was shown that by very small penalties in static displacement and frequencies the design was significantly improved in terms of the mode shapes. In this case, $k_\delta = 10{,}000$, $k_\omega = 10$, and $k_\phi = 1$ were used in the optimization. However, the values of the individual weighting factors depend strongly on the magnitude of each associated objective. Now, also the second bending mode was very well represented, and the largest relative deviation was instead observed for the torsional mode, but now as low as 16%. Again, very small deviations from the desired static properties and frequencies were observed.

An interesting observation was that the optimization did not converge to an acceptable solution from an arbitrary initial guess. The initial guess based on the sequential design solution was necessary to obtain the improved result. In general, a reasonable initial guess is required for the optimization to converge to an acceptable solution. Optimization problems typically appear when closely spaced modes changes order during optimization. During this study, this problem appeared (for modes 2 and 3) and was solved by automatically sorting the mode shapes, and frequencies, using a simple criteria based on the amount of torsional displacement near the wing root. This method proved to be more succesfull than sorting the modes using the modal assurance criterion.[17]

Although not considered during optimization, the center of gravity of the optimizied design was only about 2 cm (3%) from the scaled full-size location. The total mass of the model was about 6% higher than the desired value.

Experimental Verification

A composite internal wing beam was manufactured according to the result from the integrated design, and lead masses were added to the wing sections at the proper locations. The model was assembled and mounted in the low-speed wind-tunnel at the Royal Institute of Technology for validation testing.[14]

First, the stiffness properties of the model were investigated by applying the reference load cases using a balance mounted on a rig in the wind tunnel. The out-of-plane deformations for each applied load were measured using an optical method based on photogrammetry.[18,19] Passive reflecting markers were used as targets and attached to the model at the reference points (Fig. 1). From this investigation, it was concluded that the model was slightly stiffer than predicted. When loaded at the tip, the model showed about 18% smaller tip deflection than expected. After investigations of the wing beam, it was concluded that this was caused by the prepreg carbon fiber/epoxy tapes being somewhat thicker than expected from earlier material testing.

The dynamic testing started with identification of the frequencies of the three considered modes using an accelerometer and impact excitation. Despite the slightly stiffer structure, the natural frequencies did not differ too much from the desired frequencies. The first bending frequency was 5.0 Hz compared to the desired value of 4.6 Hz. To measure the mode shapes, the structure was excited using electromagnetic shakers. The model was excited at each resonance, and the motion of the reference points was sampled at 240 Hz during 10 s using the optical system. The discrete Fourier transform (see Ref. 20) was then used to get the amplitude and phase of the motion at each reference point. This method for mode shape extraction is referred to as the peak-amplitude method.[17] Provided that the modes are well separated, this method gives a good approximation of the true mode shapes.

The first bending mode of the model was found to be reasonably close to the desired mode shape with a maximum deviation of about 13%. The second bending mode showed somewhat larger deviations from the desired mode shape, as shown in Fig. 2.

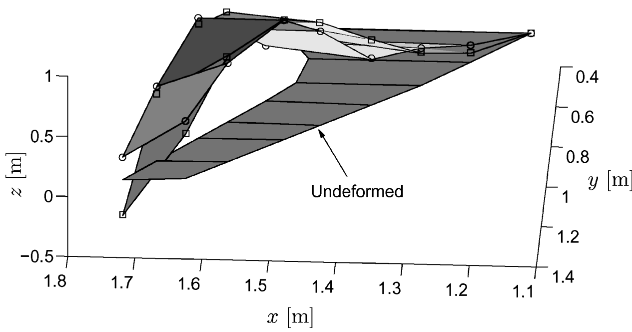

Fig. 2 Normalized displacements for mode 2: □, desired shape and ○, experimental shape.

The largest relative deviation was observed for the rear tip marker (50%). The overall shape of mode 2 was, however, well represented, and the deviations were most likely caused by the increased stiffness of the wing. Mode 3, the first torsional mode, provided some difficulties during the measurements, as discussed in more detail in Ref. 14.

Although the model showed rather good agreement with the desired behavior, the agreement was further improved by adjusting the model with respect to the measured stiffness. By model updating and redesign of the mass design variables, the largest relative deviation in mode shapes was reduced from 50 to 38%. Moreover, the corresponding result for the frequencies was a reduction from 10 to 4%.

Conclusions

The proposed design approach was successfully used to design an elastic wind-tunnel model with prescribed static and dynamic behavior. The sequential stiffness and mass design approach was first used to find a design with reasonably representative properties. The integrated stiffness and mass design optimization was then shown to improve the quality of the model significantly.

The stiffness and frequency objectives did not cause any significant trouble, whereas the mode shape matching was found to be more involved. However, the integrated approach was found to be very useful, especially when considering the mode shapes. A good mode shape match is very important because the mode shapes couple to the aerodynamic loads and, hence, stongly affect the dynamic aeroelastic behavior of the model.

A test structure was manufactured to verify the method. Although some deviations from the desired behavior were observed, the experimental testing verified the usefulness of the applied design method. The agreement was further improved by performing a model update and redesign of the tuning masses. Although the need for experienced craftsmen and engineers for aeroelastic model design still is high, numerical optimization methods are concluded to serve as very valuable tools during the design process.

Acknowledgments

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